Ranking with Long-Term Constraints

Joint work with: Zhichong Fang, Sarah Dean, and Thorsten Joachims

Recommendation Platform problem

 \times

Problem:

benefits.

0000



Recommendation Platform macro-level and micro-level

Research Question:

goals?

Micro-level Controls of Platf

Metrics: engagement through clicks, purchases, etc.

Interventions: ranking, push-notifications, etc.





Overa

$$\sum_{t=1}^{T} u(a_t | x_t) - q$$

- **Context** X_t
- **Ranking -** a_t
 - **Utility -** $u(a_t | x_t)$
- **Exposure -** $c(a_t | x_t)$
 - Target τ
- maximize (1) s.t. (2)







Problem:

t=1

the final time step T and we can not make intermediate decisions.

- **Context** X_t **Ranking -** a_t
 - **Utility -** $u(a_t | x_t)$

The overall objective can only be computed after

Violation Cost - ϕ



$\int u(a_t | x_t)$ (1) t = 1

t=1

- **Context** X_t
- **Ranking -** a_t
 - **Utility -** $u(a_t | x_t)$
- **Exposure -** $C(a_t | x_t)$
 - Target τ

$\sum c(a_t \mid x_t) \ge \tau \quad (2)$ t = 1

Overall Objective $\sum_{t=1}^{T} u(a_t | x_t) - \phi^T \left(\tau - \sum_{t=1}^{T} c(a_t | x_t) \right)$ *t*=1

Violation Cost - ϕ^T



Macro/Micro Contro

hiective overview

Observe:

At time *t* we only know the contexts x_1, \ldots, x_t , but not the future contexts x_{t+1}, \ldots, x_T .

This means we make decisions under **partial information** because we cannot evaluate our constraint.

t=1

Problem



Overall Objective

$$\phi^T \left(\tau - \sum_{t=1}^T c(a_t | x_T) \right)_+$$

Violation Cost -



Myopic Controller (MC)

$$\sum_{t=1}^{T} u(a_{t} \mid x_{t}) - \phi^{\top} \left(\frac{t}{T} \tau - \sum_{t=1}^{T} c(a_{t} \mid x_{t}) \right)_{+}$$
$$\sum_{t'=1}^{t} u(a_{t'} \mid x_{t'}) - \phi^{\top} \left(\frac{t}{T} \tau - \sum_{t'=1}^{t} c(a_{t'} \mid x_{t'}) \right)_{+}$$
$$\operatorname{argmax}_{a} u(a \mid x_{t}) - \phi^{\top} \left(\frac{t}{T} \tau - s_{t-1} - c(a \mid x_{t}) \right)_{+}$$

$$s_{t-1} = \sum_{t'=0}^{t-1} c(a_{t'} | x_{t'})$$



Define an intermediate objective at each time t

Scale the target linearly

Create intermediate objectives that do not depend on future

Remove terms that do not affect argmax

 s_{t-1} : past exposure

Problem:

The cost vector ϕ^T still depends on T because it is the final cost violation not the intermediate cost violation.

This is bad because we **suffer the maximum cost** for every intermediate constraint violation.

$$\underset{a}{\operatorname{argmax}} u\left(a \mid x_{t}\right) - \phi^{\top} \left(\frac{t}{T}\tau - s_{t-1} - c\left(a \mid x_{t}\right)\right)_{+}$$

$$s_{t-1} = \sum_{t'=0}^{t-1} c(a_{t'} | x_{t'})$$



Define an intermediate objective at each time t

Scale the target linearly

Create intermediate objectives that do not depend on future

Remove terms that do not affect argmax

 s_{t-1} : past exposure

Stationary-Controller (SC)

$$\sum_{t=1}^{T} u\left(a_{t} \mid x_{t}\right) - \phi^{\mathsf{T}}\left(\tau - \sum_{t=1}^{T} c\left(a_{t} \mid x_{t}\right)\right)$$
$$\min_{0 \le \lambda \le \phi} \frac{1}{T} \sum_{t=1}^{T} u\left(a_{t} \mid x_{t}\right) - \lambda^{\mathsf{T}}\left(\frac{1}{T}\tau - \frac{1}{T} \sum_{t=1}^{T} c\left(a_{t} \mid x_{t}\right)\right)$$

Algorithm:

Step 1argmax
$$u (a \mid x_t) - \lambda_{t-1}^{\top} \left(\frac{1}{T} \tau - c (a \mid x_t) \right)$$
Step 2 $\lambda_t = \Gamma_{[0,\phi]} \left[\lambda_{t-1} + \gamma \left(\frac{1}{T} \tau - c (a \mid x_t) \right) \right]$

Define an intermediate objective at each time t

Define an intermediate cost vector at each time t

 ϕ is bounded between 0 and ϕ

intermediate cost vector - λ is bounded between 0 and ϕ

Remove terms that do not affect argmax

Update our intermediate cost vector

Experimental Results

- KuiaRec Dataset

- Collected from the recommendation logs of the video-sharing mobile app Kuaishou
- Stationary data (not-time dependent)

- Tv Audience Dataset

- Television watching behavior of users
- Non-stationary data (time dependent)

- Metrics

- Utility nDCG
- Exposure Reciprocal Rank

Experimental Results







Predictive Controller (PC)

$$\sum_{t=1}^{T} u\left(a_{t} \mid x_{t}\right) - \phi^{\top} \left(\tau - \sum_{t=1}^{T} c\left(a_{t} \mid x_{t}\right)\right)$$
$$\min_{0 \le \lambda \le \phi} \frac{1}{T} \sum_{t=1}^{T} u\left(a_{t} \mid x_{t}\right) - \lambda^{\top} \left(\tau - \left[s_{t-1} + c\left(a_{t} \mid x_{t}\right)\right]\right)$$





Predictive Controller (PC)

$$\min_{0 \le \lambda \le \phi} \frac{1}{T} \sum_{t=1}^{T} u\left(a_t \mid x_t\right) - \lambda^{\top} \left(\tau - \left[s_{t-1} + c\left(a_t \mid x_t\right) + C\left(A_t \mid C_t\right)\right]\right)_+$$

estimate $\hat{C}(A_t | C_t)$

Step 1argmax
$$u (a \mid x_t) - \lambda_{t-1}^{\top} c (a \mid x_t)$$
 a a $\lambda_t = \Gamma_{[0,\phi]} \left[\lambda_{t-1} + \gamma \left(\tau - [s_{t-1} + \phi] \right) \right]$

Algorithm:

Define an intermediate objective at each time t

Define an intermediate cost vector at each time t

Model future contexts

using offline data

Remove terms that do not affect argmax

 $c(a_t | x_t) + \hat{C}_t \left(A_t | C_t \right)] \Big)$

Update our intermediate cost vector





Experimental Results



Cost-Aware Predictive-Controller

$$\sum_{t=1}^{T} r_t^{\mathsf{T}} \Sigma_t u - \phi^T \left(\tau - \sum_{t=1}^{T} W \Sigma_t e \right)$$

objective:

$$\min_{0 \le \lambda \le \phi} \frac{1}{T} \sum_{t=1}^{T} r_t^{\mathsf{T}} \Sigma_t u - \lambda^{\mathsf{T}} \Big(\tau - \Big[s_{t-1} + W \Sigma_t e - y \Big] \Big) \Big) = 0$$

estimate $\hat{C}(A_t | C_t)$: $\arg \max_{(\Sigma^1, \dots, \Sigma^B) \in \Pi}$

iterative objective: Step 1 $\Sigma_t = \arg \max_{\Sigma} r_t^{\mathsf{T}} \Sigma u - \Sigma$

Step 2
$$\lambda_t^b = \Gamma_{[0,\phi]} \left[\lambda_{t-1}^b + \gamma \left(\tau - [s_{t-1} + c(a_t | x_t) + \hat{C}_t^b] \right) \right], \ b = 1, \dots, E$$

$W \in \mathbb{R}^{m \times n}$	W _{ij}
$\Sigma \in \mathbb{R}^{n \times n}$	$\Sigma_{kj} = 1$
$e \in \mathbb{R}^n$	
$u \in \mathbb{R}^n$	
$r \in \mathbb{R}^n$	

contribution item j to constraint i

ranking matrix

linear exposure utility vector

linear utility vector

relevance vector

+

 $+ \hat{C}(A_t | C_t) \bigg] \bigg)_+$

. . .

using offline data

$$-\frac{1}{B}\sum_{b=1}^{B} \left(\lambda_{t-1}^{b}\right)^{\mathsf{T}} W\Sigma e$$

Break the objective into incremental rankings





Macro/Micro Control Problem control-loop

- Input: target
- Input: violation cost Φ
- TInput: final time
- Input: Π controller
- $s_t = \mathbf{0}$ **Initialize :** state



For t = 1 to T do $x_t = (\mathbf{q}_t, r_t) \sim P_t$

τ

$$\sum_{t=1}^{T} u(a_t | x_t) - \phi^T \left(\tau - \sum_{t=1}^{T} c(a_t | x_T) \right)$$



Controllers for Ranking matrix notation

Micro-level Metric
(e.g. DCG)
$$u(a_t | x_t) = r_t^{\mathsf{T}} \Sigma_t u$$

 $W \in \mathbb{R}^{m \times n} \quad \mathcal{W}_{ij}$ $\Sigma \in \mathbb{R}^{n \times n} \quad \Sigma_{kj} = 1$ $e \in \mathbb{R}^{n}$ $u \in \mathbb{R}^{n}$ $r \in \mathbb{R}^{n}$

$$\sum_{t=1}^{T} r_t^{\mathsf{T}} \Sigma_t u - \phi^T$$

 $\sum_{t=1}^{T} u(a_t | x_t) - \phi^T \left(\tau - \sum_{t=1}^{T} c(a_t | x_T) \right)_{\perp}$

contribution item j to constraint i

ranking matrix

linear exposure vector

linear utility vector

relevance vector

 $T\left(\tau - \sum W\Sigma_t e\right)$ t=1

Macro-level Metric (e.g. exposure) $c(a_t | x_t) = W \Sigma_t e$

